جريدة الدستور

Revision

And

Rules

Second secondary

Mathematics

Prepare by

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Rules

The general term (n^{th} term) of the arithmetic sequence

is $T_n = a + (n-1)d$ where n is the order of the term.

If (T_n) is an arithmetic sequence whose first term is (a) and the common difference is (d), then the general form of the arithmetic sequence is $(a, a+d, a+2d, a+3d, \dots)$

To find the order of the term whose value is x, we put $T_n = x$

To find the order of the first term whose value is less than a given value x, we put $T_n < x$

To find the order of the first term whose value is greater than a given value x, we put $T_n > x$

To find the order of the first positive term in an arithmetic sequence, we put $T_n > 0$

To find the order of the first negative term in an arithmetic sequence, we put $T_n < 0$

If the sum of three numbers in an arithmetic sequence is given, *it is better to put them* in the form $(a-d, a, a+d)$

If the sum of four numbers in an arithmetic sequence is given, *it is better to put them* in the form $(a-3d, a-d, a+d, a+3d)$

If a, b and c are three consecutive terms of an arithmetic sequence, then the middle term b equals the arithmetic mean of the two other terms a and c *i.e.* $b = \frac{a+c}{2}$ or $2b = a+c$

If T_x, T_y are two terms of an arithmetic sequence where $x \neq y$ then: d (the common difference of the sequence) $= \frac{T_y - T_x}{y - x}$

i.e. $d = \frac{T_6 - T_3}{6-3} = \frac{20-11}{6-3} = 3$

The sum of (n) terms of an arithmetic sequence in terms of its first term (a) and last term (ℓ) is $S_n = \frac{n}{2}[a + \ell]$

The sum of (n) terms of an arithmetic sequence in terms of its first term (a) and its common difference (d) is: $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n - S_{n-1} = T_n$ $\quad S_{10} - S_9 = T_{10}$

The general term (n^{th} term) of the geometric sequence

If (T_n) is a geometric sequence whose first term is (a) and its common ratio is (r), then its general term is in the form $T_n = a r^{n-1}$ where n is the order of the term.

r (the common ratio of the geometric sequence) $= \frac{\text{the value of any term in it}}{\text{the value of the preceding term directly}}$

The general form of the geometric sequence

Putting n = 1, 2, 3, ···· in the previous general term we get the general form of the geometric sequence, that is: $(a, ar, ar^2, ar^3, \dots)$

If T_x, T_y are two terms of a geometric sequence, then $r^{x-y} = \frac{T_x}{T_y}$

The difference between two squares: $1 - r^2 = (1-r)(1+r)$

The difference between two cubes: $1 - r^3 = (1-r)(1+r+r^2)$

The sum of two cubes: $1 + r^3 = (1+r)(1-r+r^2)$

$1 + r^2 + r^4 = (1-r+r^2)(1+r+r^2)$

If a, b and c are three successive terms of a geometric sequence, then b is known as the geometric mean between the two numbers a and c where: $\frac{b}{a} = \frac{c}{b}$

i.e. $b^2 = ac$, then $b = \pm\sqrt{ac}$

The arithmetic mean between two different positive real numbers is greater than their geometric mean.

Finding the sum of n terms of a geometric series in terms of its first term (a) and common ratio (r):

$S_n = \frac{a(1-r^n)}{1-r}, r \neq 1$

Finding the sum of n terms of a geometric series in terms of its first term (a) and last term (ℓ)

$\therefore S_n = \frac{a - \ell r}{1-r}, r \neq 1$

$T_n = S_n - S_{n-1}$ $\quad T_3 = S_3 - S_2$

The function V (h) is called the variation function of f at $x = x_1$

$V(h) = f(x_1 + h) - f(x_1)$

The average rate of change function

$A(h) = \frac{V(h)}{h} = \frac{f(x_1+h) - f(x_1)}{h}$ the average rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The rate of change of the function f at $x_1 = \lim_{h \to 0} A(h) = \lim_{h \to 0} \frac{f(x_1+h) - f(x_1)}{h}$

Rules of differentiation

If $f(x) = a$ where a is a real number, then $\acute{f}(x) = 0$

If $f(x) = x^n$ where $n \in \mathbb{R}$, then $\acute{f}(x) = n x^{n-1}$

If f and g are two differentiable functions with respect to x and $y = f(x) \times g(x)$, then $\frac{dy}{dx} = f(x) \times \acute{g}(x) + g(x) \times \acute{f}(x)$

i.e. $\frac{dy}{dx}$ = The first × derivative of the second + the second × derivative of the first

If f and g are two differentiable functions with respect to x and $y = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$, then:

$\frac{dy}{dx} = \frac{g(x) \times \acute{f}(x) - f(x) \times \acute{g}(x)}{(g(x))^2}$

i.e. $\frac{dy}{dx} = \frac{(\text{denominator} \times \text{derivative of numerator}) - (\text{numerator} \times \text{derivative of denominator})}{(\text{denominator})^2}$

Chain rule $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

If $y = [f(x)]^n$ where f is differentiable function with respect to x, $n \in \mathbb{R}$, then $\frac{dy}{dx} = n[f(x)]^{n-1} \times \acute{f}(x)$

i.e. Derivative of: $(\text{bracket})^n = n\,(\text{bracket})^{n-1} \times$ derivative of what inside the bracket

If $y = \sqrt{f(x)}$, then $\frac{dy}{dx} = \frac{1}{2\sqrt{f(x)}} \times \acute{f}(x) = \frac{\acute{f}(x)}{2\sqrt{f(x)}}$

i.e. $\frac{dy}{dx} = \frac{1}{2 \times \text{the root}} \times$ derivative of what inside the root.

The slope of the straight line whose equation: $ax + by + c = 0$ is $\frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$

The slope of the straight line passing through the two points (x_1, y_1), (x_2, y_2) equals $\frac{y_2 - y_1}{x_2 - x_1}$

The slope of the straight line = $\tan\theta$ where (θ) is the measure of the positive angle which the straight line makes with the positive direction of the x-axis

The slope of the x-axis = the slope of any horizontal straight line (parallel to x-axis) = zero

The slope of the y-axis = the slope of any vertical straight line (parallel to y-axis) = $\frac{1}{\text{zero}}$ (undefined)

If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively, then

(1) $L_1 // L_2 \Leftrightarrow m_1 = m_2$

(2) $L_1 \perp L_2 \Leftrightarrow m_1 \times m_2 = -1$

The equation of the straight line

Given a point on it (x_1, y_1) and the slope (m) is $y - y_1 = m(x - x_1)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

, $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

, $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

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The factorial of a positive integer n is written as $\lfloor n$ and equals the product of all the positive integers which are less than or equal to n

i.e. $\lfloor n = n(n-1)(n-2)\cdots\times 3\times 2\times 1$

and the number of factors of the factorial = n factors

$\lfloor 0 = \lfloor 1 = 1$ *So*, if $\lfloor n = 1$, then n = 0 or n = 1

The factorial of a number can be written in terms of another number less than the original one *i.e.* $\lfloor n = n\lfloor n-1 = n(n-1)\lfloor n-2 = \cdots$ where $n \in \mathbb{Z}^+$

The number of permutations of n different objects taking r at a time is denoted by the symbol ${}^nP_r$ where :

(1) ${}^nP_r = n(n-1)(n-2)\ldots(n-r+1)$ where $1 \le r \le n$, n, ${}^nP_r \in \mathbb{Z}^+$

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r} \qquad {}^nP_0 = 1 \qquad {}^nP_n = \lfloor n$$

Arrangement of n objects in one row

The number of ways to arrange n objects in one row = $\lfloor n$

Arrangement of n objects on a circle

The number of ways to arrange n objects on a circle = $\lfloor n-1$

• If n, r $\in \mathbb{N}$, $r \le n$, then :

(1) ${}^nC_r = \frac{{}^nP_r}{\lfloor r} = \frac{\lfloor n}{\lfloor r\,\lfloor n-r}$

(3) ${}^nC_n = {}^nC_0 = 1$, ${}^nC_1 = n$

(2) ${}^nC_r = {}^nC_{n-r}$ "reducing law"

(4) If ${}^nC_x = {}^nC_y$, then : $x = y$ or $x + y = n$

If the number of sides of a geometrical figure = n sides

, then the number of all line segments that represented in the figure = ${}^nC_2$

, ∵ the diagonal of the geometrical figure is the line segment joining between 2 non-consecutive vertices Mr / Mortagy

∴ The number of the diagonals of the geometrical figure

= the number of all line segments − the number of sides in the figure = ${}^nC_2 - n$

The number of diagonals of the triangle = ${}^3C_2 - 3 = 0$

, the number of diagonals of the quadrilateral = ${}^4C_2 - 4 = 2$

, the number of diagonals of the pentagon = ${}^5C_2 - 5 = 5$

, the number of diagonals of the hexagon = ${}^6C_2 - 6 = 9$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$, where tan A is defined , $\tan^2 A \ne 1$

Remember that

• The area of the triangle = $\frac{1}{2}$ its base length × the corresponding height

• The area of the triangle = $\frac{1}{2}$ the product of two side lengths × sine of the included angle between them.

Heron's formula to find area of a triangle

Let a , b and c be the side lengths of the triangle ABC , and 2 P be the perimeter of the triangle

i.e. 2 P = a + b + c , then The area of the triagnle ABC = $\sqrt{P(P-a)(P-b)(P-c)}$

If the radius length of the inscribed circle of a triangle = r ,

the area of the trianlge = Δ and the perimeter of the triangle = 2 P , then $r = \frac{\Delta}{P}$

If $y = \cos x$, then $\frac{dy}{dx} = -\sin x$

If $y = \sin x$, then $\frac{dy}{dx} = \cos x$

If $y = \tan x$, then $\frac{dy}{dx} = \sec^2 x$

(1) $\int \sin x\,dx = -\cos x + C$

(2) $\int \cos x\,dx = \sin x + C$

(3) $\int \sec^2 x\,dx = \tan x + C$

where C is an arbitrary constant.

(1) $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$

(2) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$

(3) $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$

where C is an arbitrary constant.

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta$$

The sine rule

In any Δ ABC : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

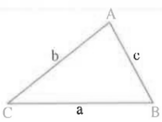
The cosine rule

any triangle ABC :

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = c^2 + a^2 - 2ca\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$



$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

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Answer the following :

1. $\int \frac{x^2+3x}{x} \, dx =$ ....

(a) $x+3$ (b) $\frac{1}{2}x^2+3x+c$ (c) x^2+3x+c (d) $\frac{x^3+3x^2}{x^2}$

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2. The solution set of the equation ${}^{11}C_r = {}^{11}C_{2r+2}$ is ....

(a) 3 (b) – 3 (c) ± 3 (d) 6

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3. The sum of the first term and fourth term in a decreasing geometric sequence = 70
The sum of the second and third terms = 60 , find the sum of infinite terms starting from its first term.

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4. If a , b , c , d , e are positive numbers forming a geometric sequence , then the geometric mean of these terms is ....

(a) c (b) $\sqrt{a\,b\,c\,d\,e}$ (c) – c (d) $-\sqrt{a\,b\,c\,d\,e}$

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5. The area of the triangle whose side lengths are 5 , 6 , 7 cm. equals .... cm^2.

(a) $3\sqrt{6}$ (b) $6\sqrt{6}$ (c) 15 (d) 105

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6. If (x , 7 , y) form an arithmetic sequence and ($x+2$, 5 , y – 6) form a geometric sequence , then $y - x =$ ....

(a) 3 (b) 8 (c) 11 (d) 14

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7. $\sin 75° \sin 75° - \cos 75° \cos 75° =$ ....

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) zero

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8. If A and B are two acute angles and $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$, then A + B = ...

(a) 30° (b) 60° (c) 45° (d) 75°

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9. If S_n is the sum of the first n terms from an arithmetic sequence and $S_{2n} = 3\,S_n$, then $S_{3n} : S_n =$ ....

(a) 4 (b) 6 (c) 8 (d) 10

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10. The number of ways that 5 students can sit on 7 seats in one row equals ....

(a) $\underline{|7}$ (b) $\underline{|5}$ (c) ${}^7P_5$ (d) ${}^7C_5$

11 The arithmetic sequence whose sixth term = 20 and the ratio between the fourth term and tenth term equals 4 : 7 , then the sum of the first fifteen terms started from its third term = ……………

(a) 360 (b) 380 (c) 400 (d) 420

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12 If $\sin x + \cos x = \sqrt{2}$, then $\sin 2x =$ ……………

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) zero

---

13 If the average rate of change in f equals 2.4 when x changes from 4 to 4.2 , then the variation in $f =$ ……………

(a) 0.32 (b) 0.48 (c) 3.6 (d) 7.2

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14 The geometric sequence whose first term is a and its common ratio r is decreasing if ……………

(a) $a > 0 , -1 < r < 0$ (b) $a > 0 , 0 < r < 1$

(c) $a < 0 , -1 < r < 0$ (d) $a < 0 , 0 < r < 1$

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15 If $y = \tan x$, then $\frac{dy}{dx} =$ ……………

(a) $1 + y$ (b) $1 - y$ (c) $1 + y^2$ (d) $1 - y^2$

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16 The function $f : f(x) = \begin{cases} x^2 + 2x , & x \le 1 \\ 4x - 1 , & x > 1 \end{cases}$ is …………… at $x = 1$

(a) continuous but not differentiable. (b) continuous and differentiable

(c) not continuous and not differentiable (d) not continuous but differentiable

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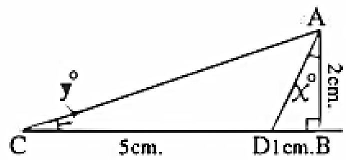
17 **In the opposite figure :**

ABC is a right-angled triangle at B

, prove that : $x + y = 45°$



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18 If $|\underline{n+1} = 30\ |\underline{n-1}$, then n = ……………

(a) 5 (b) 6 (c) 29 (d) 30

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19 If ${}^{n+1}C_{n-1} = 36$ and ${}^{x}P_3 = 120$, find $|\underline{3n - 4x}$

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20 If $x^2 + y^2 = 9 + 2xy$, then $\frac{dy}{dx} =$ ……………

(a) -1 (b) 1 (c) $\frac{x + y}{y - x}$ (d) $\frac{x - y}{x + y}$

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21 $\int (2x - 5)^6 \, dx =$ …………… + c

(a) $(x^2 - 5x)^6$ (b) $\frac{1}{14}(2x - 5)^7$ (c) $\frac{1}{14}(x^2 - 5x)^7$ (d) $\frac{1}{2}(2x - 5)^7$

22 The slope of the tangent to the curve $y = 3x^2 + 2x + 1$ at $x = 2$ equals ....

(a) 5 (b) 8 (c) 14 (d) 17

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23 If $f(3 - 2x) = 3x^2 + 1$, then $\acute{f}(7) =$ ....

(a) -12 (b) -2 (c) 6 (d) 42

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24 If $\sin(A + B) = \frac{56}{65}$, $\sin(A - B) = \frac{-16}{65}$, then $\sin A \cos B =$ ....

(a) $\frac{5}{13}$ (b) $\frac{4}{13}$ (c) $\frac{7}{13}$ (d) $\frac{-5}{13}$

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25 If 5 geometric means are inserted between a and b, then the third mean is ....

(a) $a^{\frac{1}{5}} b^{\frac{4}{5}}$ (b) $\frac{ab}{2}$ (c) $\sqrt{ab}$ (d) $a^{\frac{4}{5}} b^{\frac{1}{5}}$

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26 The n^{th} term of an arithmetic sequence $= m^2$ and the m^{th} term $= n^2$, then the common difference of the sequence d = ...

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27 The number of terms of the geometric sequence (5, 10, 20, ..., 1280) equals .... terms.

(a) 8 (b) 7 (c) 10 (d) 9

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28 If $\lfloor n = a$, then $\lfloor n - 1 =$ ....

(a) $a - 1$ (b) $n a$ (c) $n + a$ (d) $\frac{a}{n}$

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29 $\int \cos(3x + 1)\, dx = a \sin(3x + 1) + c$, then a = ...

(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{9}$

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30 The first term of a geometric sequence equals the sum of the next infinite terms, then the common ratio of this sequence equals ....

(a) 0.5 (b) 0.333 (c) 0.25 (d) 0.666

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31 Find the equation of the tangent to the curve of the function $f : f(x) = \frac{1}{x + 1}$ at the point (0, 1) which lies on it.

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32 The first term of an arithmetic sequence = 5, $T_{n+1} = T_n + 3$, then the fifth term = ...

(a) 12 (b) 20 (c) 17 (d) 19

Answer

1 (b) **2** (a)

3

$T_1 + T_4 = 70$

$\because a + ar^3 = 70$

$\therefore a(1 + r^3) = 70 \qquad (1)$

$, \because T_2 + T_3 = 60$

$\therefore ar + ar^2 = 60 \qquad \therefore ar(1 + r) = 60 \qquad (2)$

From (1) , (2) :

$\therefore \frac{a(1+r^3)}{ar(1+r)} = \frac{70}{60}$

$\frac{a(1+r)(1-r+r^2)}{ar(1+r)} = \frac{7}{6}$

$\therefore 6(1 - r + r^2) = 7r$

$\therefore 6r^2 - 13r + 6 = 0$

$\therefore (2r - 3)(3r - 2) = 0$

$\therefore r = \frac{3}{2}$ (refused because the sequence is decreasing)

or $r = \frac{2}{3}$

From (1) : $\therefore a = \frac{70}{1 + \left(\frac{2}{3}\right)^3} = 54$

$\therefore s_\infty = \frac{a}{1-r} = \frac{54}{1 - \frac{2}{3}} = 162$

4 (a) **5** (b) **6** (b) **7** (b)

8 (c) **9** (b) **10** (c) **11** (d)

12 (a) **13** (b) **14** (b) **15** (c)

16 (b)

17

$\because \tan x = \frac{1}{2} \quad , \quad \tan y = \frac{2}{6} = \frac{1}{3}$

$\therefore \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = 1$

$\therefore x + y = 45°$

18 (a)

19

$\because {}^{n+1}C_{n-1} = {}^{n+1}C_2 = 36$

$\therefore \frac{{}^{n+1}P_2}{\underline{|2}} = 36 \qquad \therefore {}^{n+1}P_2 = 72 = 9 \times 8 = {}^9P_2$

$\therefore n + 1 = 9 \qquad \therefore n = 8$

$, \because {}^xP_3 = 120 = 6 \times 5 \times 4 = {}^6P_3$

$\therefore x = 6$

$\therefore \underline{|3n - 4x} = \underline{|24 - 24} = \underline{|0} = 1$

20 (b) **21** (b) **22** (d) **23** (c)

24 (b) **25** (c) **26** (c) **27** (d)

28 (d) **29** (b) **30** (a)

31

$\because f(x) = \frac{1}{x+1} \quad , \quad \therefore \hat{f}(x) = \frac{-1}{(x+1)^2}$

$\therefore$ The slope of tangent at the point (0 , 1) equals -1

$\therefore$ The equation of tangent at the point (0 , 1) is :

$\frac{y - 1}{x - 0} = -1$

i.e. $x + y = 1$

32 (c)